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# The Wavelet Method Applied to the Analysis of Evoked Potentials.

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**Theo**

In den letzten Jahren hat die Wavelet-Transformation für Signalanalysen in vielen naturwissenschaftlichen Bereichen eine sehr große Bedeutung erlangt. Auch im Bereich der (neuro)physiologischen Grundlagenforschung läßt sich bei der Analyse von biologischen Signalen diese von Grossmann und Morlet seit 1982 neu entwickelte Methode [6] nutzbringend anwenden.

In diesem Infoblatt wird eine Methodik zur Anwendung der Wavelet-Analyse [8] vorgestellt, wie sie am Institut für Physiologie zur Untersuchung von EEG-Potentialen (evoked potentials) auf dem Hauptinstitutsrechner realisiert und bereits für verschiedene Untersuchungen in mehreren Projekten angewandt wurde. Das vollständige Literaturverzeichnis ist in [9] zu finden.

## The Wavelet

We are using the wavelet transform as a very efficient method for obtaining two-dimensional pictures of a multiscale (an inside) view of sampled signals (EEG potentials).

The general idea of a wavelet used in the wavelet transform of signals (Grossmann and Morlet, 1984; Mallat, 1989; Mallat and Zhong, 1989) comprise all complex square integrable functions $f(x)$ of finite energy, if their Fourier transform $F(\omega)$ is differentiable and fulfils the *admissibility condition*, which reads in Fourier space

$$c_f = 2\pi \cdot \int_{\omega=0}^{+\infty} \frac{|F(\omega)|^2}{|\omega|}\, d\omega < \infty. \qquad (1)$$

This condition essentially means that $f(x)$ is of zero mean. It implies that $F(0) = 0$, $F(\pm\infty) = 0$, and

$$\int_{x=-\infty}^{+\infty} f(x)\; dx = 0. \qquad (2)$$

More recently Koendering and van Doorn (1990) recommended using the term »*wavelet*« for the n-th order derivative of the Gaussian function $g(x)$ with variance $\sigma$ and mean value $\mu$:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2}, \qquad (3)$$

which reads in the normalized form with $\mu = 0$:

$$h(x) = \frac{g(x)}{g(0)} = e^{-x^2/2\sigma^2}. \qquad (4)$$

We selected as the analysing wavelet the negative second-order derivative of the normalized Gaussian function of time $h(t)$ with respect to $t$, which is called the »*Mexican hat*« function:

$$\psi_\sigma(t) = -h''(t) = \frac{1}{\sigma^2} \cdot \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{1}{2}\cdot\frac{t^2}{\sigma^2}}. \qquad (5)$$

The basic wavelet $\psi(t)$ in fig. 1, i.e. the unshifted and undilated wavelet function, is given for $\sigma = 1$ by

$$\psi(t) = -h''(t)\big|_{\sigma=1} = \left(1 - t^2\right) \cdot e^{-t^2/2}. \qquad (6)$$

As for analysing wavelets in general required, eq. (5) and (6) fulfil the admissibility condition (1) and the so-called *compatibility condition* (2). These two conditions mean that the wavelet should oscillate like a short wave and has no DC-component.

From the chosen analysing wavelet (6) we generated the other members of the wavelet family by translating $\psi_\sigma(t)$ by $\tau$ with $\tau > 0$ and dilating $\psi_\sigma(t)$ by $\sigma$, thus obtaining the collection $\psi_{\tau,\sigma}$ of the other wavelets

$$\psi_{\tau,\sigma}(t) = \psi\left(\frac{t-\tau}{\sigma}\right). \qquad (7)$$

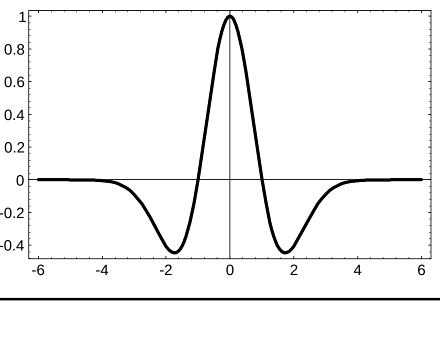


*Fig. 1: The basic wavelet $\psi(t)$ used: the »Mexican hat« function.*

## The Transform

The continuous wavelet transform $W(\tau,\sigma)$ of a real signal $e(t)$ with respect to the chosen real analysing wavelet $\psi(t)$ – in general $\psi(t)$ will be a complex function – is defined by the two-dimensional function

$$W(\tau,\sigma) = \frac{1}{\sigma^{d/2}} \int_{t=-\infty}^{+\infty} e(t) \cdot \psi\left(\frac{t-\tau}{\sigma}\right) dt \qquad (8)$$

with $\tau,\sigma \in \mathbf{R}$ and $\tau > 0$, where $\sigma^{-d/2}$ is the norm in the vector space of measurable, square-integrable $d$-dimensional functions. In this Hilbert space $\boldsymbol{L}^2(\mathbf{R}^d)$ eq. (8) is equivalent to the scalar (inner) product of the signal $e(t)$ with $\psi_{\tau,\sigma}(t)$. Each inner product can be interpreted as the convolutions between $e(t)$ and the set of $\psi(t/\sigma)$ in the time domain.

$$W(\tau,\sigma) = \left\langle e(t) \,\middle|\, \frac{1}{\sigma^{d/2}} \cdot \psi\left(\frac{t-\tau}{\sigma}\right)\right\rangle = \frac{1}{\sigma^{d/2}} \cdot e(t) * \psi\left(\frac{t}{\sigma}\right) \qquad (9)$$

Hereby is $t$ the time of the evoked potential $e(t)$ analysed and $\tau$ the shift between $e(t)$ and the wavelet $\psi_{\tau,\sigma}(t)$ applied while computing the convolution integral.

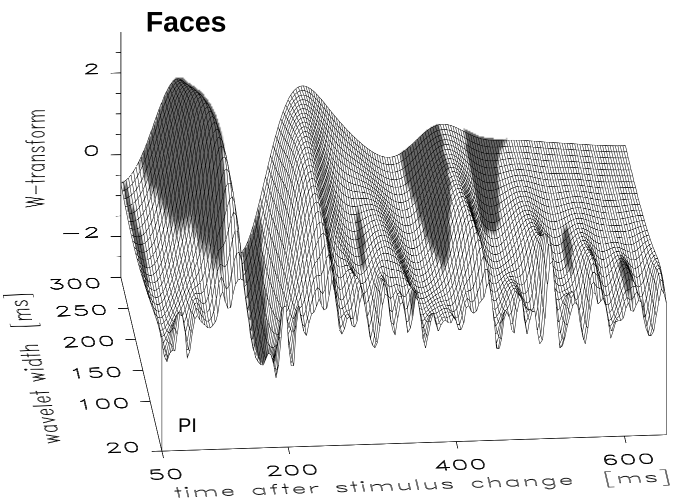


*Fig. 2: Sample of a W-diagram: the wavelet transform $W(\tau,\sigma)$ of the averaged evoked potential from fig. 3. (Taken from [9] fig. 5a).*

## The Algorithm

In real (computer) world however, one works with sampled signals obtained from $e(t)$ by measurements at the instants $t_k = k\,\Delta\tau$, where $1/\Delta\tau$ is the sampling frequency (200 Hz). Therefore, eq. (8) must be replaced by its discrete version with $e[n]$ as the sampled sequence of the EEG potential $e(t)$.

$$W(k\,\Delta\tau,\sigma) = \frac{\Delta\tau}{\sigma^{d/2}} \cdot \sum_{n=0}^{N-1} e[n] \cdot \psi\left[\frac{(n-k)\,\Delta\tau}{\sigma}\right] \qquad (10)$$

with $\sigma \in \mathbf{R}^+$ and $n, k \in \mathbf{Z}$. $N$ is the number of sampled points in $e[n]$, which might be upsampled with interpolation by a factor $\alpha$ (we use $\alpha = 5$). The exponent $d$ was set at 3. The generation process of the wavelet family was done by translating $\psi[m]$ by $\tau$ = 50–650 ms in 1 ms increments, and dilating $\psi[m]$ by $\sigma$ = 20–300 ms in 10 ms increments, thus creating 29 wavelet transforms of the sampled evoked potential $e[n]$.

In our package ***WaveEEG*** the discrete wavelet transform is implemented as a fast algorithm of the discrete convolution [5] using the sampled wavelet $\psi[m]$ of half-width $M$ as a convolution filter (kernel), i.e. $\psi[m]$ has $(2\,M + 1)$ elements with $M < \alpha\,N/2$. For a symmetric kernel the computer algorithm of the discrete convolution using an upsampled signal $e[n,\alpha]$ with $n = 1,\ldots,\alpha\,N$ is given by

$$W(k\,\Delta\tau,\sigma) = \chi \cdot \sum_{i=0}^{2M} e_{n+i+1-M} \cdot \psi_i \qquad (11)$$

for all $(M + 1 \le n < \alpha\,N - M)$, otherwise $W = 0$, by which $\chi$ is a convenient scaling factor.

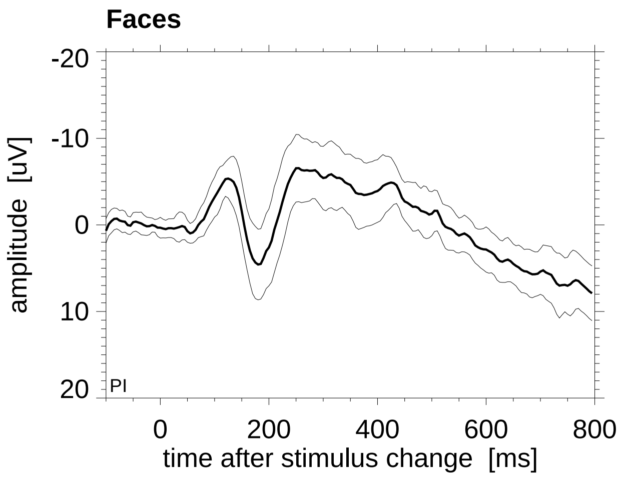


*Fig. 3: Sample of an averaged evoked potential recorded through electrodes F4–T6 with ±3 s.e. range shown. The stimulus categorie is »faces«. (Taken from* [9] *fig. 2a).*

We computed the discrete convolutions of the wavelets with the sampled evoked potential (fig. 3) using the developed program ***WaveEEG***, under which the resulting wavelet transform $W(\tau,\sigma)$ depended on the time $t$ and on the width $\sigma$ of the wavelet function as described. The shift $\tau$ in eq. (9) is the convolution parameter. The function $\psi(t\text{–}\tau)$ is the mirrored $\psi(t)$ at the vertical line $\tau = t/2$ (»*Faltung*«).

For the selected set of wavelets (for which the width $\sigma$ was varied systematically) the wavelet transforms $W(\tau,\sigma)$ were computed, yielding a two-dimensional representation of the evoked potential. This is displayed in a three-dimensional *W-diagram* (fig. 2), showing a continuous measure of frequency-specific components of the evoked potentials over time.

An analysing wavelet (used as a convolution filter) based on the Gaussian function has the best possible simultaneous concentration in time and in frequency space [7]. Varying $\sigma$ of eq. (10) corresponded besides the application of a variable temporal frequency filter. According to the term with $\sigma$ in the nominator of eq. (8) it is evident, the smaller $\sigma$, the higher the gain and the smaller the bandwidth of the convolution filter, and vice-versa.

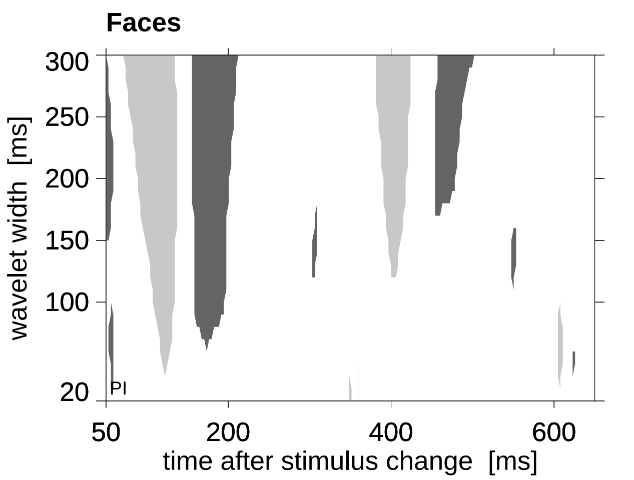


*Fig. 4: The contour plot of fig. 2. The shaded areas correspond to ranges of the W-diagram significantly below or above the chosen “reference model”, i.e. the wavelet transform of a set of randomly shuffled evoked potentials. (Taken from* [9] *fig. 7a).*

## Contour Plots

In evaluating the W-diagrams we developed a reliable method of computing the statistical significance of the peaks and troughs appearing in the W-diagrams. Using the wavelet transform of 30 different sets of randomly shuffled averaged evoked potentials as a »reference model«, a statistical comparison at the ±3 s.d. limit was computed. The results were displayed in two-dimensional contour plots (C-plots). A sample C-plot is shown in fig. 4. More informations on this topic and further illustrations are given in [9].

## The Equipment

For performing the wavelet analysis, a well-tuned computer equipment with a perfect graphic display and printing capabilities is needed. The computer must be able of processing large-sized arrays of data in a reasonable time. Approximately 48 MByte of (virtual) memory was consumed by this wavelet application.

We used a small-sized VAX cluster (running the virtual multitasking system VMS) configured with a server (VAX-3500), a microVAX-II with two 1 GByte disks, a colour workstation (VAX-3200, running X-Windows), a Macintosh computer (Mac-II), and a PostScript laser printer. All systems were netted together by Ethernet.

The prechecking of optimal algorithms was done on the Macintosh Computer under *Mathematica* [1] using the excellent GeorgiaTech package *SignalProcessing* [2]. The final programming for production on the VAX computers was implemented under *IDL*, the Interactive Data Language [3], a 4-th generation language designed especially for signal processing purposes. This language supports directly the discrete convolution of signals with a kernel, and the Fourier transform. In the last years *IDL* becomes a standard for high-quality scientific visualization on different computer platforms.

The graphical outputs (plots) were all written in the *PostScript* language [4]. For binding and resizing the plots into documents we used the portable representation as Encapsulated PostScript (EPS files). This was done on the Macintosh Computer using the Aldus *PageMaker*, a layout program, and Aldus *FreeHand*, a graphic tool respectively.

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